The partitioned solution of Ref. 2 uses smaller matrices than the $[Z^{-1}]$ technique of Ref. 1. However, it becomes necessary to choose in advance which of the α are to be surplus and which can safely be put in (1, 1) correspondence with the nodal deflections. This problem does not arise with the $[Z^{-1}]$ technique.

Arithmetically it is not yet clear which is the better. Methods of inversion are known which take full advantage of symmetry, using just over $\frac{1}{2}$ N^2 words of storage and $\frac{1}{2}$ N^3 multiplications to invert an $N \times N$ matrix. A modified Waugh and Dwyer technique achieves this, as does the row-inversion technique discussed in Ref. 3. These techniques are normally recommended only for positive definite matrices. It is the authors' feeling that such a technique is applicable with a modified order of pivoting. If such a technique is applicable, the partitioned solution will use only slightly fewer operations than the $[Z^{-1}]$ solution. Nothing is yet known concerning the relative numerical accuracy.

References

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Comments on "Sputtering in the Upper Atmosphere"

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Weight losses from a gold surface orbiting at 200 km are found by McKeown et al.,[1] the angle between the surface and the direction of motion being 30°. The authors ascribe these weight losses to sputtering of gold by N_2, the impact energy being 9 ev, and calculate an erosion rate of 0.1 Å/day or about 5×10^8 Au atoms/sec-cm². In view of results of sputtering-yield studies at very low bombarding ion energies,[2] it seems doubtful that any detectable sputtering would occur at 9 ev. It seems more likely that the weight losses observed are caused by outgassing. For N_2 outgassing, the observed weight losses would correspond to an outgassing rate of 10^{-10} torr-liters/sec-cm², which is very low for an unbaked surface.

Aside from this, since the authors indicate that they consider their results to be an upper limit on sputtering from a satellite surface, it is well to point out an apparent arithmetical error. The sputtering yield may be calculated from

$$Y = n/Nv \sin\theta$$

where

$n = 5 \times 10^8$ Au atoms/sec-cm²
$N = 7.82 \times 10^9$ N_2/cm³ (Ref. 3)
v = impact velocity $= 8 \times 10^5$ cm/sec
$\theta = 30°$

These data give $Y = 1.6 \times 10^{-7}$ Au/N_2, essentially an order of magnitude lower than the value given by McKeown et al.

References

[1] McKeown, D., Fox, M. G., Schmidt, J. J., and Hopper, D., "Sputtering in the upper atmosphere," AIAA J. 2, 400–401 (1964).

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[2] Stuart, R. V. and Wehner, G. K., "Sputtering yields at very low bombarding ion energies," J. Appl. Phys. 33, 2345–2352 (1962).

[3] *U. S. Standard Atmosphere* (U. S. Government Printing Office, Washington, D. C., 1962), p. 81. We follow the assumption by McKeown et al. that the composition in the upper atmosphere is mainly N_2.

Reply by Authors to R. V. Stuart

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Stuart's comment on outgassing has been previously considered in the referenced papers[1,2] in which it was concluded that the measured weight loss was due to sputtering. The weight loss was attributed to sputtering and not to out-gassing for the following reasons. First, in making an erosion measurement, not one but four gold surfaces were exposed to the vacuum of space. The gold surfaces were plated on two matched quartz oscillator crystals. Only one of the plated surfaces was actually under molecular bombardment. A photograph of the erosion gage is shown in one of the referenced articles.[2] The output of the gage is the beat frequency of the two crystals. Any outgassing should produce equal mass losses from all four of the gold surfaces, and the frequency of both crystals will increase. This frequency increase is cancelled at the gage output since only the beat frequency is telemetered. The mass change sensed by the gage will be that produced by particles impacting on the surface exposed to the molecular stream.

To check this assumption, a control gage with gold-plated crystals as well as a test gage with gold-plated crystals were flown on Discoverer 26.[1] Mass measurements were not taken until the satellite was in orbit for four days to permit the crystals to outgas. The output of the two gages was monitored closely for the following two days. The results of the test were referenced.[2] The maximum error in measuring the thickness of surface eroded that could be attributed to variation in the power supply voltage, temperature changes, outgassing, and any other unknown cause was found to be ±0.05 Å/day. The erosion rate reported in Ref. 4 was given then as 0.1 ± 0.05 Å/day, and it was reasonable to assume it was caused by sputtering from the results of the previous work.

In calculation of the sputtering yield an apparent error is present if one considers that a satellite has a circular orbit. Satellites do not. Since the atmospheric density drops off rapidly with altitude (between 200 and 330 km, the atmospheric density decreases by an order of magnitude), an average density must be used in calculating the yield. As a result of this density variation, a satellite generates a molecular beam that is intensity modulated as it passes between perigee and apogee.

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Unfortunately, the orbital parameters of the two satellite flights reported in the note were classified, and only a vague statement that the satellites "orbited at *about* 200 km above the earth" could be given. For those who would be interested in checking our calculation, we cited an earlier article[2] where approximately the same yield for gold was determined at normal incidence. Here it was stated that Discoverer 26 had a perigee of 230 km and an apogee of 810 km. Its period was 95 min, of which approximately 20 min was spent in the denser atmosphere near perigee. If the relationship given here to find the yield had been used, no confusion would have resulted, since the formula includes a bombardment factor for the time the satellite spends near perigee.

Stuart's calculation on the sputtering yield reduces to ours by defining N as the average density, which in this case was 1×10^9 N_2/cm^3, and $Y \sim 1 \times 10^{-6}$ Au/N_2, as was given in our note.

We also consider it unwise for Stuart to compare his laboratory results with those made in the upper atmosphere and conclude it is doubtful that any detectable sputtering would occur at 9 ev because the experimental conditions are different.[3] In his paper, Stuart states his detector had a yield threshold of 10^{-5} atom/ion, and with it he would not have been able to measure lower yields at any energy. The beams are different. He was working with noble gas plasmas, and the upper atmosphere is composed mainly of neutral molecules moving in free molecular flow. In the upper atmosphere the vacuum is clean. He stated that his vacuum contained impurities at a partial pressure of 10^{-6} torr and a monolayer could build up on a clean surface in a second. Assuming that one started with a clean surface at a certain instant in time, it is difficult to imagine an experimenter madly twisting dials to get his apparatus working properly so that a good yield measurement even in the 10^{-5} range could be accomplished within a second. The measurement has to be made in less than a second because his beam was capable of sputtering about 5×10^{-3} monolayer/sec when the yield is 10^{-5}; otherwise, the clean surface would become contaminated before the beam could sputter enough surface atoms to permit a measurement to be made.

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[1] McKeown, D., Fox, M. G., and Schmidt, J. J., "Measurement of surface erosion from Discoverer 26," ARS J. **32**, 954–955 (1962).

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Erratum: "A Model for the Transition Regime in Hypersonic Rarefied Gasdynamics"

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[AIAA J. **2**, 1047–1054 (1964)]

On page 1048 of the original article, the first line in the left column ("assumptions will become progressively invalid when the") should be the first line on page 1049. When the page proof was released, the line was in its correct place but was inadvertently transposed when the printer was making up the forms.

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